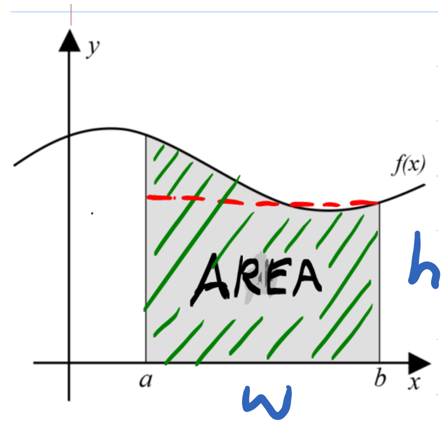
The General Case of a Riemann Sum….

Once upon a time there was a function. A Sasquatch wanted to find the area under the function below from **a** to **b**. He decided to approximate by using the area of a rectangle.

As you can see from the diagram we could use the

Following formula:



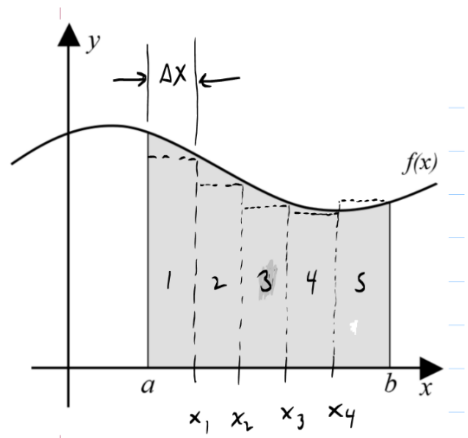
$$\text{AERA} = \text{WIDTH} \times \text{height}$$

$$\text{AERA} = w \times h$$

$$\text{AREA} = (b-a)\cdot f(b)$$

This is a poor approximation so the Sasquatch tried using more rectangles. He used 5 rectangles all with equal widths Δx.

He then found the area to be:



$$\text{AREA} = \underbrace{\Delta x}_{\text{WIDTH}}\underbrace{\left[f(x_1)\right]}_{\text{HEIGHT}} + \Delta x\left[f(x_2)\right] + \Delta x\left[f(x_3)\right] + \Delta x\left[f(x_4)\right] + \Delta x\left[f(x_5)\right]$$

The Sasquatch noticed that the height of each rectangle was different but the width **Δx** was always the same so he factored **Δx** out:

$$\text{AREA} = \Delta x\left[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)\right]$$

An Alien from outer space saw the Sasquatch's work and said "Hey you can write that as a ***summation***!"

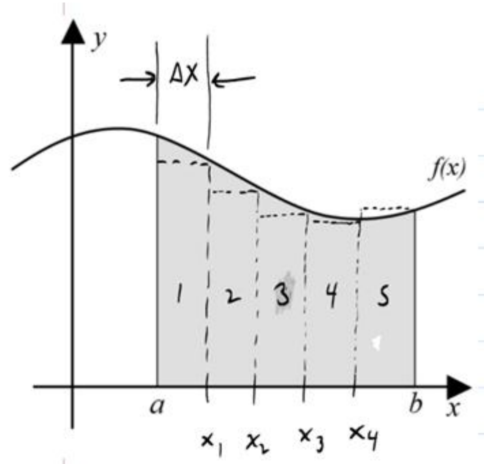
$$A = \sum_{i=1}^{n} f(x_i)\,\Delta x$$

$$A = \sum_{i=1}^{n} f(x_i)\,\Delta x$$

Great Said the Sasquach!..

But what what is x_i ?

Well...If you look carefully you will see that x_i keeps changing as we advance i in the summation. The formula for x_i would be:



$$x_i = a + \Delta x(i)$$

What is Δx ?

Δx just depends on the length of the interval and how many rectangles we use.

$$\Delta x = \frac{b-a}{n}$$

n = # of RECTANGLES YOU WISH TO USE.

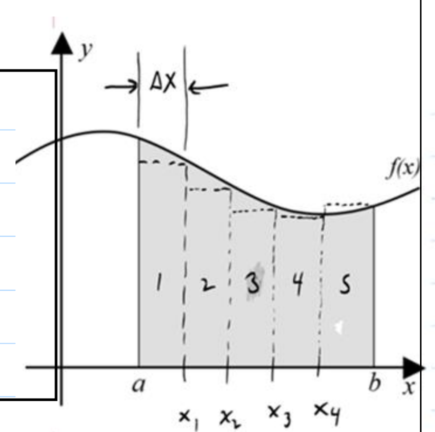
a → b INTERVAL OF AREA.

Great. Now we have a formula that can approximate the area under **any** function ***f(x)*** between the intervals **a** and **b**

$$\boxed{A = \sum_{i=1}^{n} f(x_i)\,\Delta x}$$

$$x_i = a + \Delta x(i)$$

$$\Delta x = \frac{b-a}{n}$$



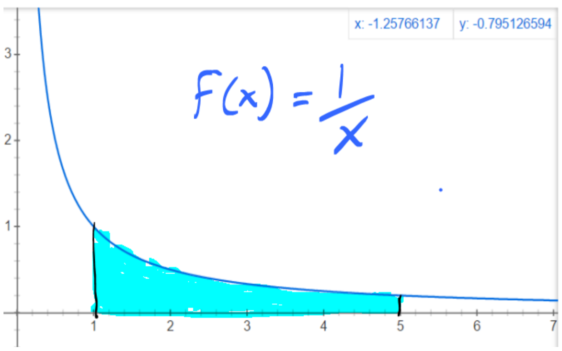
Let's call it a **Riemann Sum**. Let's practice using it!

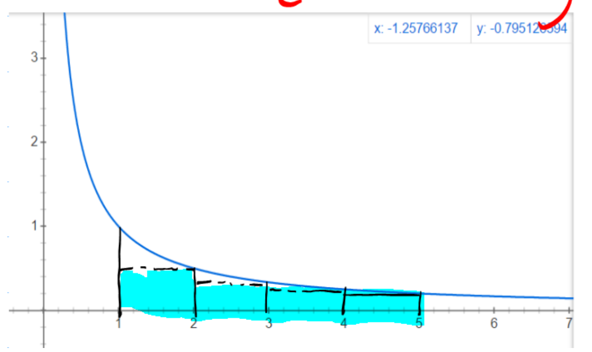
Examples:

EXAMPLE ESTIMATE THE AREA UNDER THE FUNCTION:

$f(x) = \frac{1}{x}$ FROM $x = 1$ to $x = 5$

USE $n = 4$ {# of RECTANGLES}





Using Riemann Sum Formula:

$$A = \sum_{i=1}^{n} f(x_i)\,\Delta x \qquad x_i = a + \Delta x(i)$$

$$\Delta x = \frac{b-a}{n}$$

$$A = \sum_{i=1}^{n} f(x_i)\,\Delta x$$

- $\Delta x = \frac{b-a}{n}$

$$A = \sum_{i=1}^{4} f\big(a + \Delta x(i)\big)\,\Delta x$$

- $\Delta x = 1$
- $x_i = a + \Delta x(i)$

$$A = \sum_{i=1}^{4} f\big(1 + 1(i)\big)\,1$$

REMINDER: $f(x) = \frac{1}{x}$

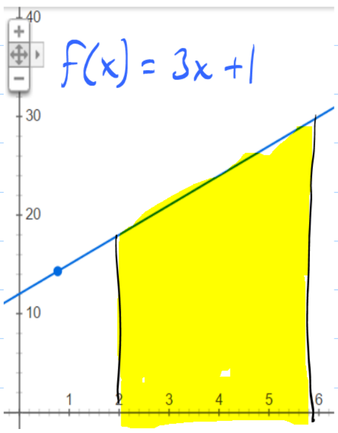
$$A = \left[\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right] 1$$

$$A = 0.5 + 0.333 + 0.25 + 0.2$$

$$A = 1.2835$$

THIS IS THE APPROXIMATE AREA UNDER THE FUNCTION $f(x) = \frac{1}{x}$ FROM $1 \to 5$

EX. 2



FIND THE AREA UNDER $3x + 1$ FROM $2 \to 6$ USING 4 RECTANGLES!

$$A = \sum_{i=1}^{4} f(x_i)\,\Delta x$$

$$A = \sum_{i=1}^{4} f\big(a + \Delta x(i)\big)\,\Delta x$$

$$A = \sum_{i=1}^{4} f\big(2 + 1(i)\big) \cdot 1$$

- $n = 4$
- $\Delta x = \frac{b - a}{n}$
- $\Delta x = 1$
- $x_i = a + \Delta x(i)$

$f(x) = 3x + 1$

$$A = [f(3) + f(4) + f(5) + f(6)] \cdot 1$$

$$A = [10 + 13 + 16 + 19] \cdot 1$$

$$A = 58 \text{ units}^2$$

Example #3

FIND AREA UNDER $y = x^2$ FROM $\underset{a}{0} \to \underset{b}{1}$

$$\boxed{\text{USE } n = 4}$$

- $\Delta x = \frac{b-a}{n}$
- $\Delta x = \frac{1}{4}$
- $x_i = a + \Delta x(i)$

$f(x) = x^2$

$$A = \sum_{i=1}^{4} f\big(a + \Delta x(i)\big)\Delta x$$

$$A = \sum_{i=1}^{4} f\left[0 + \frac{1}{4}(i)\right]\frac{1}{4}$$

$$A = \left[\left(\frac{1}{4}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{3}{4}\right)^2 + (1)^2\right]\cdot\frac{1}{4}$$

$$A = 0.46875$$