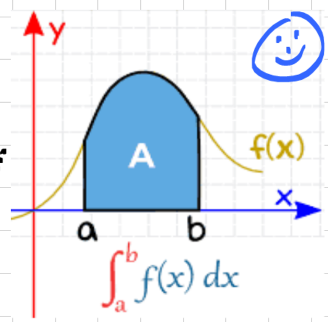
Finding the Antiderivative (Integral) of a function

AS WE MOVE FORWARD PLEASE REMEMBER THAT THE REASON WE ARE DOING THIS INTEGRALS (OR ANTIDERIVATIVES) IS.... WE WILL EVENTUALLY USE THEM TO FIND AREA UNDER FUNCTIONS



SIMPLE INTEGRAL EXAMPLES

Title

DO YOU UNDERSTAND THAT:

INTEGRATION IS "UNDOING" OF DERIVATION

INTEGRATION = ANTIDERIVATIVE

$$\int f'(x)\,dx = f(x)$$

if $f(x) = x^2$

$f'(x) = 2x$

$\Downarrow$

$$\int 2x\,dx = x^2$$

An important tool:

THE QUESTION IS: WHAT FUNCTION (IF I DO THE DERIVATIVE OF IT) WILL GIVE ME THE ORIGINAL FUNCTION.

$$\int x^2\,dx = \frac{1}{3}x^3 + C$$

$$\int x^3\,dx = \frac{1}{4}x^4 + C$$

$$\int x^n\,dx = \frac{1}{(n+1)}x^{n+1} + C$$

THE QUESTION IS: WHAT FUNCTION (IF I DO THE DERIVATIVE OF IT) WILL GIVE ME THE ORIGINAL FUNCTION.

① $\int x^4\,dx = \frac{1}{5}x^5 + C$ $\qquad \frac{1}{3/2} = \frac{2}{3}$

② $\int x^9\,dx = \frac{1}{10}x^{10} + C$ $\qquad \frac{1}{2} + 1 = \frac{3}{2}$

③ $\int \sqrt{x} = \int x^{1/2} = \frac{2}{3}x^{3/2} + C$

④ $\int \frac{1}{\sqrt{x}} = \int x^{-1/2} = 2x^{1/2} + C$ $\qquad -\frac{1}{2} + 1 = \frac{1}{2}$

⑤ $\int \frac{1}{5}x^2 = \frac{1}{15}x^3 + C$ $\qquad \frac{1}{(1/2)} = 2$

⑥ $\int 5x^3 + 2x + 1 + \sqrt{x}$

$= \frac{5}{4}x^4 + x^2 + x + \frac{2}{3}x^{3/2} + C$

You Try:

Answer on next pg.

(7) $\int -6\sqrt{x}\ dx$

(8) $\int e^{4x}\ dx$

Note: if the derivative of the inside of "nested" function is a constant the antiderivative will have to be divided by that constant

(9) $\int 2e^{3x}\ dx = \frac{2}{3}e^{3x} + C$

(10) $\int \sin(2x)\ dx = -\frac{1}{2}\cos(2x) + C$

(11) $\int \cos(3x+1)\ dx = \frac{1}{3}\sin(3x+1) + C$

(12) $\int (3x+5)^6\ dx$

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(13) $\int (6x+\pi)^3\ dx$

Important example:

(14) $\int \frac{1}{x} = \ln x + C$

(15) $\int \frac{2x^2 + x + 2}{\sqrt{x}}\, dx$

$= \int 2x^{3/2} + x^{1/2} + 2x^{-1/2}\, dx$

$= \frac{4}{5}x^{5/2} + \frac{2}{3}x^{3/2} + 4x^{1/2} + C$

(16) $\int (x-1)(x^2+1)\, dx$

Multiply out first

(17) $\int \left(\frac{1}{x^2} + \frac{1}{x} + \sin x + e^{2x}\right) dx$

18 $\int (\sqrt{x}+x)\sqrt{x} + (3x+1)^6 + \cos(\pi x+1)\, dx$

Answers:

7 $= -4x^{3/2} + C$

8 $\frac{1}{4}e^{4x} + C$

12 $= \frac{1}{21}(3x+5)^7 + C$

13 $= \frac{1}{24}(6x+\pi)^4 + C$

16 $= \frac{1}{4}x^4 - \frac{1}{3}x^3 + \frac{1}{2}x^2 - x + C$

17 $= -x^{-1} + \ln x - \cos x + \frac{1}{2}e^{2x} + C$

18 $\frac{1}{2}x^2 + \frac{2}{5}x^{5/2} + \frac{1}{21}(3x+1)^7 + \frac{1}{\pi}\sin(\pi x+1) + C$