Review List:

Definition of a derivative
How the difference quotient was created
Using the difference quotient to find derivatives
Finding the equation of a tangent line

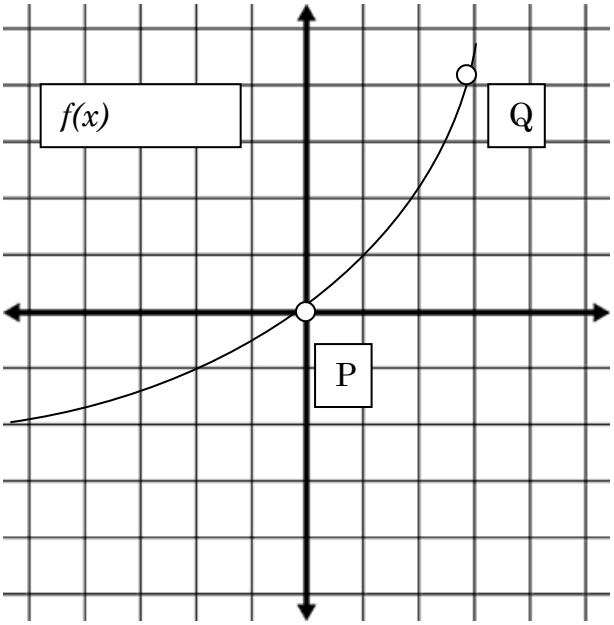
Power Rule,
Chain, Rule,
Product Rule
Quotient Rule
Exponential and log functions,
Trig functions

Implicit Differentiation
Higher Order Derivatives
Rate of change in **motion**.
Differentiability

Derivatives Self-TEST/Review 2020 Section 1

1. Describe what a derivative is:

2. a) Write down the equation for the *difference quotient formula* (remember this formula is a limit).



b) What is the *difference quotient* trying to accomplish?

b) Examine the sketch of the function *f(x)* to the right.

c) Draw a ***secant line*** through P and Q

d) Use your notes and label "*h*" is on the graph.

e) Label *f(x+h)* on the graph

f) Explain what "*h*" is in the *difference quotient formula*

e) Explain why, to get the derivative of *f(x),* we need to make *h* approach zero

4. *Use the difference quotient to find f'(x) of the following function.*
(use known derivative techniques to check your answers)

$f(x) = x^2 + 3$

5. *Find the equation of the line tangent to the function* $f(x) = 6\sqrt{x-4}$ *at the point where x=10. Use any method.*

6.

a) Fin d y' if $y = \dfrac{2}{x^2 - 2x + 3}$

b) Find y' if $y = \dfrac{\sqrt[3]{x+1}}{(1-x)}$

c) Find y' if $y = \cos e^{\sin x}$

d) Find y' if $y = \sin^3\left(\tan x^2\right)$

e) Find y' if $y = x^2 \cos x$

f) $y = 3^{-2x^4}$

g) $y = \log_5 -x^5$

7.

An object moves along a coordinate line, its position at each time $t \geq 0$ given by $x(t) = 3t^2 - 7t + 4$. Find the position, velocity, acceleration, and speed at time $t_0 = 4$.

8.

Use implicit differentiation to obtain $\frac{dy}{dx}$ in terms of x and y for $x^2 - 4xy + 2y^2 = 5$.

9.

Find the second derivative for $y = \sqrt{9 + x^3}$.

Answers:

#5 $y = \frac{3}{\sqrt{6}}x + \sqrt{6}$

6. $\frac{-4x-4}{(x^2-2x+3)^2}$

a)

b) $y' = \dfrac{2x+4}{3(x+1)^{2/3}(1-x)^2}$

c) $y' = -\sin e^{\sin x}\left(e^{\sin x}\right)\cos x$

d) $y' = 6x\sin^2\left(\tan x^2\right)\cos\left(\tan x^2\right)\sec^2 x^2$

e) $y' = x\left(2\cos x - x\sin x\right)$

f) $3^{-2x^4}\cdot(-8x^3)ln3$

g) $\dfrac{-5x^4}{-x^5\ln 5}$

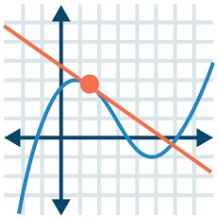
$\dfrac{dy}{dx} = \dfrac{2y-x}{2y-2x}$

$x(4) = 24$; $v(4) = 17$; $a(4) = 6$
Speed $= |v(4)| = 17$

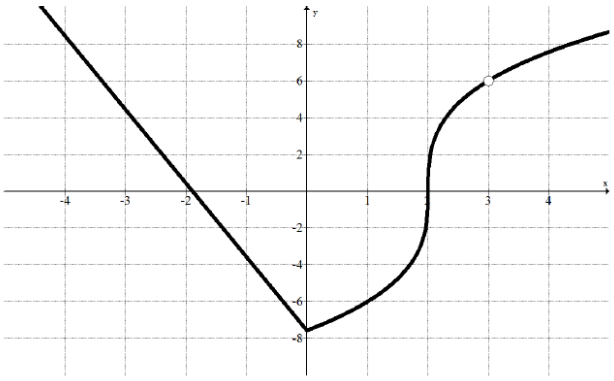
$\dfrac{3x^4 + 108x}{4(9+x^3)\sqrt{9+x^3}}$

1. $\frac{dy}{dx}$

Section 2 TEST yourself



Consider the graph of $f(x)$ shown below. Use this graph to answer questions #16 - 17.



16. **(2 pts.)** State all of the values of x at which $f(x)$ is continuous but not differentiable.

a) $x = 0, x = 2, x = 3$

b) $x = 3$

c) $x = 0, x = 3$

d) $x = 0, x = 2$

2. *Use **implicit differntiation** to find the derivative of the following function and the slope of* $2y - y^3 = xy$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

3. *Use the **difference quotient** to find f'(x) of the following function:*

$f(x) = \sqrt{2x + 4}$

4. *By sketching the following function (or using a graphing utility) determine any points where h(x) is not differentiable.*

   $h(x) = x^{\frac{2}{3}}$

5. Given the position function $s = 2t^3 - 27t^2 + 108t + 5$ (*s* in meters, t in seconds) find the following:

   a. Position of the particle at *t=0*
   b. The time when the particle has zero acceleration.
   c. The velocity when the acceleration is zero.
   d. The acceleration when the velocity is zero.
   e. The *total **distance*** traveled in the first 7 seconds.
      (you might have to graph to get the answer for e.

6. Is $G(x) = \begin{cases} Cos\, x, & if\ x \le \pi \\ x^2 - \pi x - 1, & if\ x > \pi \end{cases}$ differentiable at $x = \pi$?

Show your work and determine if G(x) is **differentiable** and **continuous** at x = π

7. Let $f(x) = \begin{cases} ax^2 + 10, & x < 2 \\ x^2 - 6x + b, & x \geq 2 \end{cases}$

If *f(x)* is **differentiable** and **continuous**, **find the correct values of a and b**.

SHOW YOUR WORK!

8. Use the alternative difference quotient to find the slope of the following function at $x = 4$

$f(x) = x^2 - 3x - 5$ *at* $x = 4$ $\qquad f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

9. Do the following on the differentiation sheet:

1-4,7,9,12,15,17,20,25,26,30,31,32,33,35,37,39,49,55,58,83,99,106,107,113,130,131,133,138

Answers:

#1 D

#2 $y'(x) = -\dfrac{y}{x + 3y^2 - 2}$

#3 Use conjugate

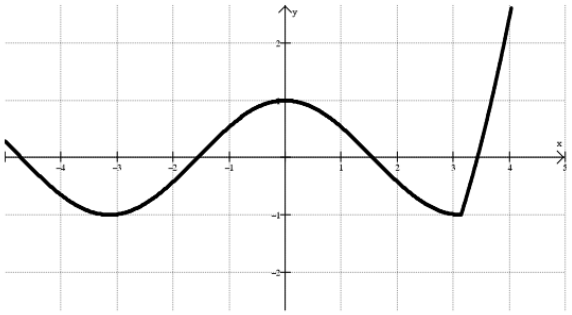
$$\frac{1}{\sqrt{2x+4}}$$

#4 *f(x) not differentiable at x =0.*

#5 a) $5m$ b) $t = 4.5s$ c) $v = -13.5 m/s$ d) $a = -18 m/s^2,\ 18 m/s^2$

e) $d = 124m$ *(displacement)*
distance is actually 173.

#6



Yes Continuous……but**, not differentiable**

#7 a=-1/2 b=16

#8 Slope = 5

#9 Answers on back of differentiation sheet