Calculus 12

The Area Between Two Curves

1. Make a rough sketch of each pair of functions, shade the area that is to be found between the curves on the given interval, and then calculate the area:

a. $f(x) = -x^2 + 6x,\ g(x) = 2x - 1,\ [1,4]$
b. $f(x) = 6cosx,\ g(x) = -1,\ \left[0,\frac{\pi}{2}\right]$
c. $f(x) = e^x,\ g(x) = e^{2x},\ [ln1, ln3]$
d. $f(x) = sinx,\ g(x) = cosx,\ \left[\frac{\pi}{4},\frac{\pi}{2}\right]$
e. $f(x) = x^2 + 2,\ g(x) = x + 8,\ [-1,2]$

2. Make a rough sketch of each pair of functions, shade the area between them, determine the intersection points, and calculate the area.

a. $f(x) = -x^2 + 6,\ g(x) = -x + 4$
b. $f(x) = 2x^2 - 6x,\ g(x) = -8x + 4$
c. $f(x) = x^2 + 3,\ g(x) = -x^2 + 5$
d. $f(x) = sinx,\ g(x) = \frac{2}{\pi}x$ $\quad Hint\ to\ find\ intersection\ points:\ A\ line\ with\ a\ slope\ of\ \frac{2}{\pi}$

$$has\ a\ slope\ of\ \frac{Rise}{Run} = \frac{2}{\pi}$$
$$= \frac{2 \div 2}{\pi \div 2}$$
$$= \frac{1}{\frac{\pi}{2}}$$
$$So\ b = 0\ and\ m = \frac{Rise}{Run} = \frac{1}{\frac{\pi}{2}}$$
$$Sketch\ y = sinx\ then\ sketch\ the\ line\ y = \frac{2}{\pi}x\ on\ top$$

Answers

1. a. $12\ units^2$ b. $\frac{12+\pi}{2}\ units^2$ c. $2\ units^2$ d. $\left(\sqrt{2}-1\right)units^2$ e. $\frac{33}{2}units^2$
2. a. $(-1,5)\ and\ (2,2)\ ;\ A = \frac{9}{2}units^2$
   b. $(-2,20)\ and\ (1,-4)\ ;\ A = 9\ units^2$
   c. $(-1,4)\ and\ (1,4);\ A = \frac{8}{3}units^2$
   d. $\left(-\frac{\pi}{2},-1\right),\ (0,0)\ and\ \left(\frac{\pi}{2},1\right)\ ;\ 2 - \frac{\pi}{2}\ units^2$