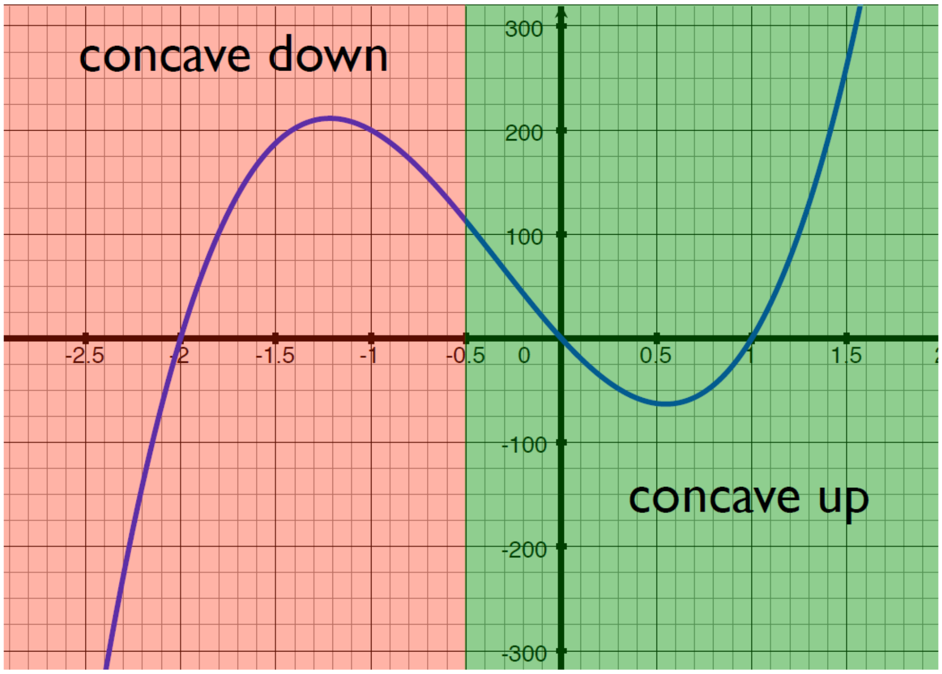
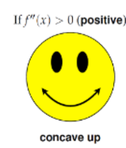
Concavity and the **Second Derivative** Test



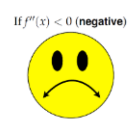
The ***second derivative*** *of a function* ***f ''(x)*** can be described as the *rate of change of the slope*

(a measure of how the **slope is changing**)

If the slope is ***increasing*** – concave up



If the slope is ***decreasing*** – concave down



Look carefully at the graphs above to see if this makes sense

Test for Concavity

Let f be a function and suppose f'' exists at all points in the interval (a, b).

Then f is **concave up** on (a, b) if $f''(c) > 0$ for all c in (a, b)

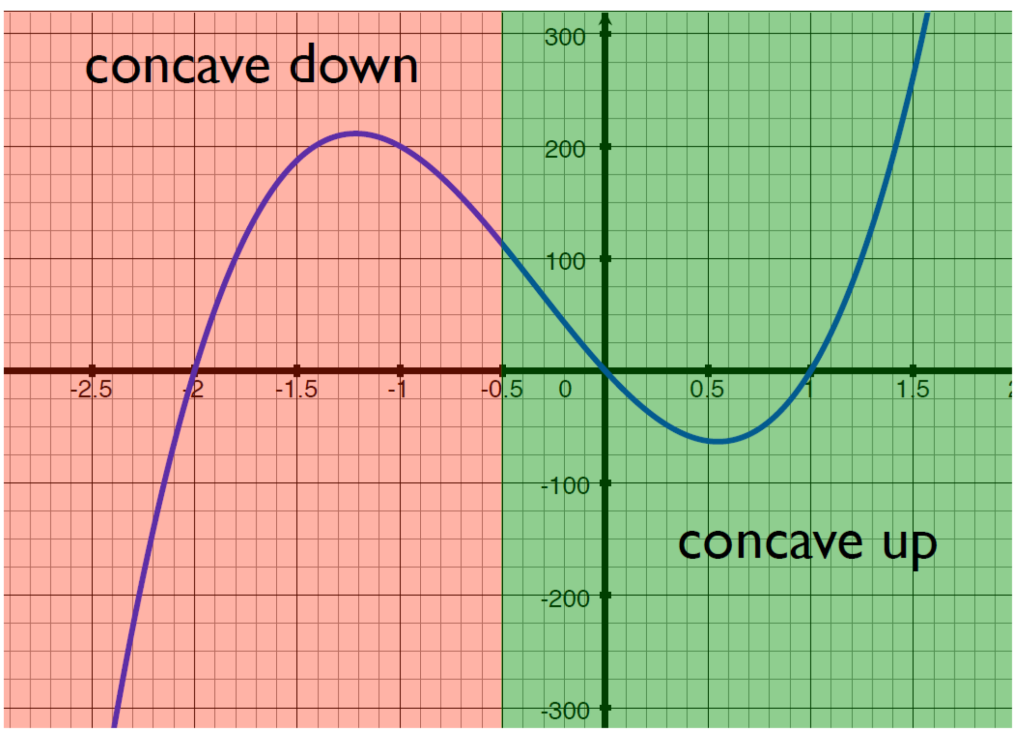
Then f is **concave down** on (a, b) if $f''(c) < 0$ for all c in (a, b).

If $f''(x) > 0$ (**positive**)

concave up

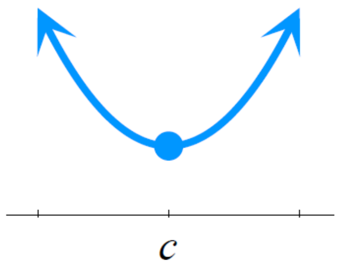
If $f''(x) < 0$ (**negative**)

concave down



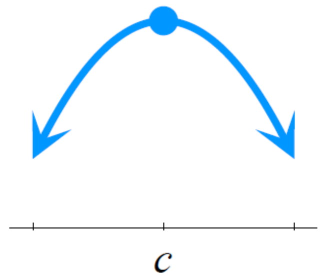
The concavity of a function can be useful in finding local extrema.

$f'(c) = 0$
$f''(c) > 0$



local minimum

$f'(c) = 0$
$f''(c) < 0$



local maximum

The Second Derivative Test

Let f be a function and c a point in the domain of f, then:

$f'(c) = 0$ and $f''(c) > 0 \implies$ $f(c)$ is a **local minimum**

$f'(c) = 0$ and $f''(c) < 0 \implies$ $f(c)$ is a **local maximum**

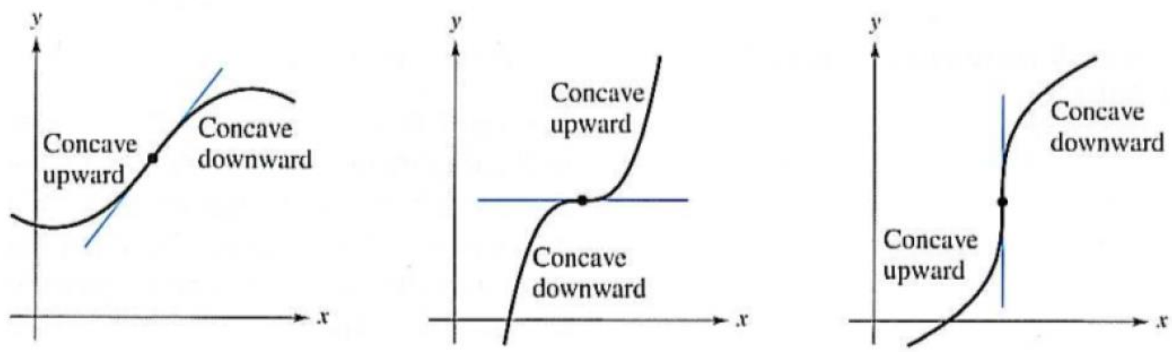
$f'(c) = 0$ and $f''(c) = 0 \implies$ **NOTHING!**

$f'(c) = 0$ and $f''(c)$ is undef. $\implies$ **NOTHING!**

Inflection points

An inflection point is were the a graph will **change** from concave up to concave down or the opposite. Inflection points occure when there is a change in concavity.

$$\text{inflection point} \quad \Rightarrow \quad f''(c) = 0 \text{ or } f''(c) \text{ is undefined}$$



Definition

An x-value where a function goes from concave up to concave down is called an **inflection point**.

$x = c$ is an inflection point $\quad \Rightarrow \quad f''(c) = 0$ or $f''(c)$ is undefined

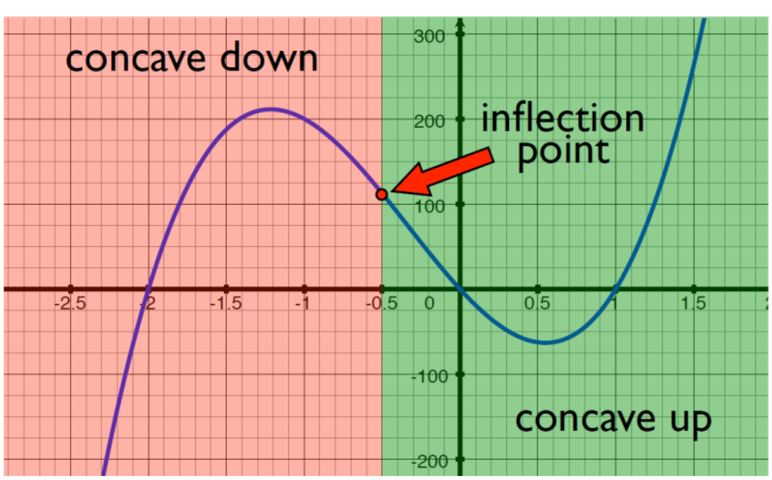
$x = c$ is an inflection point $\quad \not\Leftarrow \quad f''(c) = 0$ or $f''(c)$ is undefined

The points $x = c$ *in the domain* of f where either

(1) $f''(c) = 0$, or

(2) $f''(c)$ is undefined,

will be called 2nd **critical points** of f.



Workbook Problems:

Pg. 195 1-4, try 5 and 6

Pg. 198 Ex.3

Pg. 199 1-3

Pg. 200 1-4

Pg. 203 1-2

Pg. 210 Ex.5